Strong convergence rates of probabilistic integrators for ordinary differential equations

Tim Sullivan, FU Berlin / Zuse Institute Berlin

Probabilistic integration of a continuous dynamical system is a way of systematically introducing model error, at scales no larger than errors inroduced by standard numerical discretisation, in order to enable thorough exploration of possible responses of the system to inputs. It is thus a potentially useful approach in a number of applications such as forward uncertainty quantification, inverse problems, and data assimilation. We extend the convergence analysis of probabilistic integrators for deterministic ordinary differential equations, as proposed by Conrad et al. (Stat. Comput., 2016), to establish mean-square convergence in the uniform norm on discrete- or continuous-time solutions under relaxed regularity assumptions on the driving vector fields and their induced flows. Specifically, we show that randomised high-order integrators for globally Lipschitz flows and randomised Euler integrators for dissipative vector fields with polynomially-bounded local Lipschitz constants all have the same mean-square convergence rate as their deterministic counterparts, provided that the variance of the integration noise is not of higher order than the corresponding deterministic integrator.
This is joint work with Han Cheng Lie (FU Berlin) and Andrew Stuart (Caltech).